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D-branes in NSNS and RR pp-wave backgrounds and S-duality

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Abstract: We investigate boundary conditions for open strings in NSNS and RR pp-wave backgrounds constructed by Russo and Tseytlin, which are S-dual to each other. We show that if we do not turn on any boundary term (i.e. gauge field), D-branes in the RR background cannot move away from the origin in most cases, while those in the NSNS background can move anywhere. We construct RR counterparts of D3-branes in the NSNS background as D3-branes with gauge fluxes and show that indeed they can move anywhere, in accord with S-duality.

Keywords: Superstrings and Heterotic Strings, D-branes, Penrose limit and pp-wave background.

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1. Introduction

pp-wave backgrounds are rare examples that worldsheet theory of fundamental strings in them is exactly solvable in spite of the presence of non-zero RR flux [1, 2]. In light cone gauge worldsheet action of fundamental strings in Green-Schwarz(GS) formalism is quadratic and solvable. Closed string mass spectrum is that of a collection of infinite number of harmonic oscillators with mode numbers shifted by mass terms.

pp-wave backgrounds can be derived from AdS solutions by taking Penrose limit [3]. Limit version of AdS/CFT correspondence has been proposed in [4].

It is very important to investigate open strings and D-branes in these backgrounds for understanding string dynamics in RR backgrounds and AdS/CFT correspondence. Some such attempts have been done in [5, 6, 7] for pp-wave from $AdS_5 \times S^5$. See also [8]–[12]. Main results of [5, 6, 7] are that half BPS D-branes must sit at the origin, and less supersymmetric configurations are inconsistent.

In this paper we investigate open string boundary conditions in NSNS and RR pp-wave backgrounds given in [13]. (Some analysis has been already done in [14]. Some supergravity solutions have been constructed in [14, 15].) These two backgrounds can be obtained by taking limits of $AdS_3 \times S^3 \times T^4$, and S-dual to each other. BPS brane spectra are expected to respect S-duality. We will see that indeed BPS D3-brane spectra coincides in an interesting way.

This paper is organized as follows. In section 2 we analyze boundary conditions of open strings in the RR background without introducing any boundary term. In particular we consider the conditions which preserve half of the dynamical supersymmetry. We will find that in most cases D-branes must sit at the origin, similarly to the result in [5, 6].

In section 3 we repeat the same analysis for the NSNS background. We will find that BPS configurations are almost the same ones as in the flat background. In particular, D-branes can be anywhere.

Since the NSNS background has non-zero NSNS 2-form, it is D3-branes with non-zero gauge fields that are S-dual to D3-branes in section 3. In section 4 we construct such D3-branes in the RR background and show that indeed they can move anywhere. (Similar phenomenon in pp-wave background originating from $AdS_5 \times S^5$ has been reported in [16].) Furthermore low-lying open string spectra on BPS D3-branes coincide with those of section 3. Section 5 contains conclusions.

2. D-branes in RR pp-wave background

In this section we consider open strings in the following pp-wave background with the RR 3-form flux [13]:

$$\begin{aligned} ds^2 &= dudv - f^2 x^i x^i du^2 + dx^i dx^i + dx^a dx^a \,, \\ C_{ij} &= -\mathcal{F}_{ij} u \,, \end{aligned} \tag{2.1}$$

where C_{ij} is the RR 2-form, $\mathcal{F}_{i_1 j_1} = 2f\epsilon_{i_1 j_1}$, $\mathcal{F}_{i_2 j_2} = 2f\epsilon_{i_2 j_2}$, $i,j = 1,2,3,4$, $i_1, j_1 = 1,2$, $i_2, j_2 = 3,4$ and $a = 5,6,7,8$. It is shown in [13] that this background can be obtained as a limit of $AdS_3 \times S^3 \times T^4$. x^a correspond to T^4.

The worldsheet action of fundamental strings in GS formalism is

$$\begin{aligned} L = \frac{1}{\pi\alpha'} \Big[& \frac{1}{2}\partial_+ u \partial_- v + \frac{1}{2}\partial_+ v \partial_- u - m^2 x^i x^i + \partial_+ x^i \partial_- x^i + \partial_+ x^a \partial_- x^a + \\ & + iS_R \partial_+ S_R + iS_L \partial_- S_L + 2imS_L \gamma^1\gamma^2 R_+ S_R \Big] \,, \end{aligned} \tag{2.2}$$

where we have taken the light cone gauge, in which this action is quadratic and therefore exactly solvable, $u = 2\alpha' p^u \tau$ and $m = \alpha' p^u f$. γ^i are SO(8) 8×8 real gamma matrices[1] and $R_\pm = \frac{1}{2}(1 \mp \gamma^1\gamma^2\gamma^3\gamma^4)$.

We decompose $S_{R,L}$ into 4-component spinors $s_{R,L}$ and $\hat{s}_{R,L}$:

$$S_{R,L} = \begin{pmatrix} s_{R,L} \\ \hat{s}_{R,L} \end{pmatrix}, \tag{2.3}$$

$$R_+ S_{R,L} = \begin{pmatrix} s_{R,L} \\ 0 \end{pmatrix}, \qquad R_- S_{R,L} = \begin{pmatrix} 0 \\ \hat{s}_{R,L} \end{pmatrix}, \qquad \gamma^{12} S_{R,L} = -\begin{pmatrix} \Lambda s_{R,L} \\ \hat{\Lambda}\hat{s}_{R,L} \end{pmatrix}, \tag{2.4}$$

where Λ and $\hat{\Lambda}$ are 4×4 antisymmetric matrices with $\Lambda^2 = \hat{\Lambda}^2 = -1$. We can see only $s_{R,L}$ have the mass term. Since analysis for x^a and $\hat{s}_{R,L}$ is trivial, we will often omit them below.

This action has two types of supersymmetry i.e. dynamical supersymmetry and kinematical supersymmetry. They are given as follows.

- dynamical supersymmetry

$$\delta x^i = i\epsilon_R \gamma^i R_+ S_R + i\epsilon_L \gamma^i R_+ S_L \,, \tag{2.5}$$

$$\delta x^a = i\epsilon_R \gamma^a R_- S_R + i\epsilon_L \gamma^a R_- S_L \,, \tag{2.6}$$

$$\delta S_R = -\partial_- x^i \epsilon_R \gamma^i R_+ - \partial_- x^a \epsilon_R \gamma^a R_- - m x^i \epsilon_L \gamma^i \gamma^1 \gamma^2 R_+ \,, \tag{2.7}$$

$$\delta S_L = -\partial_+ x^i \epsilon_L \gamma^i R_+ - \partial_+ x^a \epsilon_L \gamma^a R_- - m x^i \epsilon_R \gamma^i \gamma^1 \gamma^2 R_+ \,, \tag{2.8}$$

where $R_+ \epsilon_{L,R} = 0$ i.e. $\epsilon_{L,R}$ have 4 non-zero components.

[1] For an explicit expression, [17, see page 288].

- kinematical supersymmetry

$$\begin{aligned}\tilde{\delta}S_R &= e^{-2m\gamma^1\gamma^2 R_+\tau}\tilde{\epsilon}_R + e^{2m\gamma^1\gamma^2 R_+\tau}\tilde{\epsilon}_L\,,\\ \tilde{\delta}S_L &= e^{-2m\gamma^1\gamma^2 R_+\tau}\tilde{\epsilon}_R - e^{2m\gamma^1\gamma^2 R_+\tau}\tilde{\epsilon}_L\,. \end{aligned}\tag{2.9}$$

In this section we do not introduce non-zero gauge field on D-branes. Then variation of the action is

$$\begin{aligned}\delta S = \frac{1}{\pi\alpha'}\int d^2\sigma\Big[&-2\delta x^i\left(\partial_+\partial_- x^i - m^2 x^i\right) - 2\delta x^a\partial_+\partial_- x^a + i\delta s_R\left(\partial_+ s_R - m\Lambda s_L\right)+\\ &+ i\delta s_L\left(\partial_+ s_L - m\Lambda s_R\right) + i\delta\hat{s}_R\partial_+\hat{s}_R + i\delta\hat{s}_L\partial_-\hat{s}_L +\\ &+\partial_\sigma\left\{-\frac{1}{2}\delta x^i\partial_\sigma x^i - \frac{1}{2}\delta x^a\partial_\sigma x^a + \frac{i}{2}\left(S_R\delta S_R - S_L\delta S_L\right)\right\}\Big]\,.\end{aligned}\tag{2.10}$$

From this, equations of motion and boundary condition for open strings read as follows.

equations of motion

$$\begin{aligned}\partial_+\partial_- x^i + m^2 x^i &= 0\,,\\ \partial_+ s_R - m\Lambda s_L &= 0\,,\\ \partial_- s_L - m\Lambda s_R &= 0\,.\end{aligned}\tag{2.11}$$

boundary conditions at $\sigma = 0, \pi$

$$\mathrm{N} : \partial_\sigma x^i = 0\,,\tag{2.12}$$

$$\mathrm{D} : x^i = \text{constant}\,,\tag{2.13}$$

$$S_L = M S_R\,,\qquad M^T M = 1\,.\tag{2.14}$$

Note that in the light cone gauge u and v are always Neumann.

We consider only the case where both ends of open strings satisfy the same boundary condition.

Mode expansion of x^i and its hamiltonian are as follows.

- x^i : N

$$\begin{aligned}x^i = i\sqrt{\frac{\alpha'}{2}}\Bigg[&\sqrt{\frac{2}{\omega_0}}\left(a_0^i e^{-i\omega_0\tau} - a_0^{i\dagger}e^{i\omega_0\tau}\right)+\\ &+\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\left(a_n^i e^{-i\omega_n\tau} - a_n^{i\dagger}e^{i\omega_n\tau}\right)\left(e^{in\sigma}+e^{-in\sigma}\right)\Bigg]\,,\end{aligned}\tag{2.15}$$

$$\omega_n = \mathrm{sgn}(n)\sqrt{n^2+(2m)^2}\,,\qquad (n\neq 0)\,,\qquad \omega_0 = 2m\,,\tag{2.16}$$

$$\left[a_n^i, a_{n'}^{j\dagger}\right] = \delta^{i,j}\delta_{n,n'}\,,\tag{2.17}$$

$$\begin{aligned}H &= \frac{1}{2\pi\alpha'}\int d\sigma\left[\frac{1}{2}\partial_\tau x^i\partial_\tau x^i + \frac{1}{2}\partial_\sigma x^i\partial_\sigma x^i + 2m^2 x^i x^i\right],\\ &= \frac{1}{2}\sum_{n=0}^{\infty}\omega_n\left(a_n^i a_n^{i\dagger} + a_n^{i\dagger}a_n^i\right)\,.\end{aligned}\tag{2.18}$$

- x^i : D

$$x^i = \frac{x_1^i(e^{2m\sigma}-e^{-2m\sigma}) - x_0^i(e^{2m(\sigma-\pi)}-e^{-2m(\sigma-\pi)})}{e^{2m\pi}-e^{-2m\pi}} + \\ + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\left(a_n^i e^{-i\omega_n\tau} + a_n^{i\dagger}e^{i\omega_n\tau}\right)\left(e^{in\sigma}-e^{-in\sigma}\right), \quad (2.19)$$

$$\left[a_n^i, a_{n'}^{j\dagger}\right] = \delta^{i,j}\delta_{n,n'}\,, \quad (2.20)$$

where $x_0^i = x^i(\sigma=0)$, $x_1^i = x^i(\sigma=\pi)$.

$$H = \frac{m}{2\pi\alpha'}\frac{(e^{2m\pi}+e^{-2m\pi})((x_1^i)^2+(x_0^i)^2)-4x_1^i x_0^i}{e^{2m\pi}-e^{-2m\pi}} + \frac{1}{2}\sum_{n=1}^{\infty}\omega_n\left(a_n^i a_n^{i\dagger} + a_n^{i\dagger}a_n^i\right). \quad (2.21)$$

Next we give mode expansions and hamiltonians of fermions. We assume that M is given by a product of γ^i's. Then the number of γ^i's in M must be even, since S_L and S_R have the same chirality. In addition since we assume $MR_\pm = R_\pm M$ later, we impose this condition from the beginning. We define sign factors $\eta, \xi^i = \pm 1$ as follows,

$$\gamma^i M = \xi^i M\gamma^i, M^T = \eta M\,, \quad (2.22)$$

and decompose M into 4×4 matrices:

$$MS_R = \begin{pmatrix} Ns_{R,L} \\ \hat{N}\hat{s}_{R,L} \end{pmatrix}. \quad (2.23)$$

Then, noting that $(N\Lambda)^2 = -\eta\xi_1\xi_2 = \pm 1$,

- $\eta\xi_1\xi_2 = +1$

$$s_L = \frac{\sqrt{\alpha'}}{2}\Bigg[\sqrt{2}(\cos 2m\tau + \sin 2m\tau N\Lambda)s_0 + \quad (2.24) \\ + \sum_{n=-\infty,n\neq 0}^{\infty} e^{-i\omega_n\tau}\left(\sqrt{\frac{\omega_n+n}{\omega_n}}e^{-in\sigma} - \mathrm{sgn}(n)i\sqrt{\frac{\omega_n-n}{\omega_n}}e^{in\sigma}N\Lambda\right)s_n\Bigg],$$

$$s_R = \frac{\sqrt{\alpha'}}{2}\Bigg[\sqrt{2}(\sin 2m\tau\Lambda + \eta\cos 2m\tau N)s_0 \quad (2.25) \\ + \sum_{n=-\infty,n\neq 0}^{\infty} e^{-i\omega_n\tau}\left(\eta\sqrt{\frac{\omega_n+n}{\omega_n}}e^{in\sigma}N - \mathrm{sgn}(n)i\sqrt{\frac{\omega_n-n}{\omega_n}}e^{-in\sigma}\Lambda\right)s_n\Bigg],$$

$$\left\{s_n^\alpha, s_{n'}^\beta\right\} = \delta^{\alpha,\beta}\delta_{n,n'}, s_n^\dagger = s_{-n}\,, \quad (2.26)$$

$$H = \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma[is_L\dot{s}_L + is_R\dot{s}_R] \\ = -ims_0 N\Lambda s_0 + \frac{1}{2}\sum_{n=-\infty,n\neq 0}^{\infty}\omega_n s_{-n}s_n\,. \quad (2.27)$$

- $\eta\xi_1\xi_2 = -1$

$$s_L = \sqrt{\frac{\pi\alpha' m}{\sinh(2\pi m)}}\Big[\cosh(m(2\sigma-\pi))\eta N + \sinh(m(2\sigma-\pi))\Lambda\Big]\Sigma + \tag{2.28}$$
$$+\frac{\sqrt{\alpha'}}{2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} e^{-i\omega_n\tau}\left(\sqrt{\frac{\omega_n+n}{\omega_n}}e^{-in\sigma} - \right.$$
$$\left. - \,\mathrm{sgn}(n)\frac{1}{\omega_n}\sqrt{\frac{\omega_n-n}{\omega_n}}(2m+inN\Lambda)e^{in\sigma}\right)s_n ,$$

$$s_R = \sqrt{\frac{\pi\alpha' m}{\sinh(2\pi m)}}[\cosh(m(2\sigma-\pi)) + \sinh(m(2\sigma-\pi))N\Lambda]\Sigma + \tag{2.29}$$
$$+\frac{\sqrt{\alpha'}}{2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} e^{-i\omega_n\tau}\left(\mathrm{sgn}(n)i\sqrt{\frac{\omega_n-n}{\omega_n}}e^{-in\sigma}\Lambda + \right.$$
$$\left. + \frac{1}{\omega_n}(-2mi\Lambda + \eta nN)\sqrt{\frac{\omega_n+n}{\omega_n}}e^{in\sigma}\right)s_n ,$$

$$\{s_n^\alpha, s_{n'}^\beta\} = \delta^{\alpha,\beta}\delta_{n,n'}\,, \quad s_n^\dagger = s_{-n}\,, \quad \{\Sigma^\alpha,\Sigma^\beta\} = \delta^{\alpha,\beta}\,, \quad \Sigma^\dagger = \Sigma\,, \tag{2.30}$$
$$H = \frac{1}{2}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} \omega_n s_{-n}s_n\,. \tag{2.31}$$

Now we investigate the condition that half of the dynamical supersymmetry are unbroken. From eq. (2.14) the following condition must be satisfied.

$$\delta S_L = M\delta S_R \qquad \text{at} \qquad \sigma = 0,\pi\,. \tag{2.32}$$

We require the first, second and third terms of (2.7) and (2.8) cancel separately, for each i and a. Then M must satisfy $MR_\pm = R_\pm M$, and

$$\begin{aligned} M\delta S_R = & -\eta\xi^i\partial_- x^i\left(\epsilon_R M\gamma^i R_+\right) - \eta\xi^a\partial_- x^a\left(\epsilon_R M\gamma^a R_-\right) - \\ & -\eta\xi^i\xi^1\xi^2 m x^i\left(\epsilon_L M\gamma^i\gamma^1\gamma^2 R_+\right). \end{aligned} \tag{2.33}$$

Comparing the above with (2.8), unbroken supersymmetry parameters are given in the following form.

$$\epsilon_R M = \zeta\epsilon_L\,, \qquad \zeta = \pm 1\,. \tag{2.34}$$

Equating the first terms of (2.33) and (2.8), we find the following result.

- $\zeta\eta\xi^i = +1$

  x^i must be Neumann. Then the third terms of (2.33) and (2.8) give an additional constraint: $\zeta\xi^1\xi^2\xi^i = +1$ i.e. $\eta\xi^1\xi^2 = +1$, otherwise $x^i = 0$, which contradicts the Neumann condition.

- $\zeta\eta\xi^i = -1$

  x^i must be Dirichlet. Then by the cancellation of the third terms,

1. $\zeta\xi^1\xi^2\xi^i = +1$ i.e. $\eta\xi^1\xi^2 = -1$
   Positions of D-branes can be arbitrary.
2. $\zeta\xi^1\xi^2\xi^i = -1$ i.e. $\eta\xi^1\xi^2 = +1$
   D-branes must be at $x^i = 0$

Analysis for x^a is similar, but without additional constraint coming from the third terms of (2.7) and (2.8). It is shown easily that the supersymmetry transformation of x^i vanish at the boundary if x^i is Dirichlet.

We can classify M and the result, up to symmetry rotation and $\gamma^1 \ldots \gamma^8 = 1$, is given as follows.

- $M = 1$ $(\eta = 1, \xi_1\xi_2 = 1)$
  $\zeta = +1$ $\quad x^{1,2,3,4,5,6,7,8}$: N $\quad$ (D9-brane)
  $\zeta = -1$ $\quad x^{1,2,3,4,5,6,7,8}$: D $(x^i = 0)$ $\quad$ (D1-brane)

- $M = \gamma^{13}$ $(\eta = -1, \xi_1\xi_2 = -1)$
  $\zeta = +1$ $\quad x^{1,3}$: N, $\quad x^{2,4}$: D $(x^i = 0)$, $\quad x^{5,6,7,8}$: D $\quad$ (D3-brane)
  $\zeta = -1$ $\quad x^{1,3}$: D $(x^i = 0)$, $\quad x^{2,4}$: N, $\quad x^{5,6,7,8}$: N $\quad$ (D7-brane)

- $M = \gamma^{14}$ $(\eta = -1, \xi_1\xi_2 = -1)$
  $\zeta = +1$ $\quad x^{1,4}$: N, $\quad x^{2,3}$: D $(x^i = 0)$, $\quad x^{5,6,7,8}$: D $\quad$ (D3-brane)
  $\zeta = -1$ $\quad x^{1,4}$: D $(x^i = 0)$, $\quad x^{2,3}$: N, $\quad x^{5,6,7,8}$: N $\quad$ (D7-brane)

- $M = \gamma^{56}$ $(\eta = -1, \xi_1\xi_2 = 1)$
  $\zeta = +1$ $\quad x^{1,2,3,4}$: D, $\quad x^{5,6}$: N, $\quad x^{7,8}$: D $\quad$ (D3-brane)

- $M = \gamma^{1234}$ $(\eta = 1, \xi_1\xi_2 = 1)$
  $\zeta = +1$ $\quad x^{1,2,3,4}$: D $(x^i = 0)$, $\quad x^{5,6,7,8}$: N $\quad$ (D5-brane)
  $\zeta = -1$ $\quad x^{1,2,3,4}$: N, $\quad x^{5,6,7,8}$: D $\quad$ (D5-brane)

- $M = \gamma^{1256}$ $(\eta = 1, \xi_1\xi_2 = 1)$
  $\zeta = +1$ $\quad x^{1,2}$: D $(x^i = 0)$, $\quad x^{3,4}$: N, $\quad x^{5,6}$: D, $\quad x^{7,8}$: N $\quad$ (D5-brane)
  $\zeta = -1$ $\quad x^{1,2}$: N, $\quad x^{3,4}$: D $(x^i = 0)$, $\quad x^{5,6}$: N, $\quad x^{7,8}$: D $\quad$ (D5-brane)

Note that except $M = \gamma^{56}$ case D-branes sit at $x^i = 0$. This phenomenon is similar to the result for pp-wave obtained from $AdS_5 \times S^5$ [5, 6]. $M = \gamma^{1234}$ case has been already analyzed in [14]. We note for future reference that $M = \gamma^{12}$ also gives D3-branes, though this case is not allowed by the third terms of (2.7) and (2.8).

Next let us consider the condition that some of kinematical supersymmetry are unbroken. It is given by $\tilde{\delta}S_L = M\tilde{\delta}S_R$. We decompose $\tilde{\epsilon}_{R,L}$ into 4-component spinors in the same way as (2.3):

$$\tilde{\epsilon}_{R,L} = \begin{pmatrix} \kappa_{R,L} \\ \hat{\kappa}_{R,L} \end{pmatrix} . \tag{2.35}$$

Then,

$$(1 - N)e^{2m\tau\Lambda}\kappa_R = (1 + N)e^{-2m\tau\Lambda}\kappa_L \, , \tag{2.36}$$

$$(1 - \hat{N})\hat{\kappa}_R = (1 + \hat{N})\hat{\kappa}_L \, . \tag{2.37}$$

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Noting that $\hat{N}^2 = \pm 1$ the second equation means half of $\hat{\kappa}_R$ and $\hat{\kappa}_L$ are unbroken. For the first equation we consider the following two cases separately.

- $N^2 (= \eta) = 1$

  The left and right hand side must vanish separately:

$$(1 - N) e^{2m\tau\Lambda} \kappa_R = (1 + N) e^{-2m\tau\Lambda} \kappa_L = 0\,. \tag{2.38}$$

  These equations are satisfied for arbitrary τ only when $N\Lambda = \Lambda N$. Then half of κ_R and κ_L are unbroken.

- $N^2 = -1$

  The following must be satisfied.

$$e^{2m\tau\Lambda} \kappa_R = N e^{-2m\tau\Lambda} \kappa_L\,. \tag{2.39}$$

  If $N\Lambda = -\Lambda N$ this is satisfied for arbitrary τ and half of κ_R and κ_L are unbroken. If $N\Lambda = \Lambda N$ this is not satisfied.

The above classification is summarized as follows. If $(N\Lambda)^2 (= -\eta\xi_1\xi_2) = -1$, half of κ_R and κ_L are unbroken. If $(N\Lambda)^2 = 1$, none of κ_R and κ_L are unbroken. By this analysis we find that $M = \gamma^{56}$ (and γ^{12}) preserves 1/4 of kinematical supersymmetry, and $M = 1, \gamma^{13}, \gamma^{14}, \gamma^{1234}, \gamma^{1256}$ preserve half.

3. D-branes in NSNS pp-wave background

In this section we consider D-branes on the NSNS pp-wave background which is S-dual to the RR background in the previous section [13]:

$$\begin{aligned} ds^2 &= du dv - f^2 x^i x^i du^2 + dx^i dx^i + dx^a dx^a\,, \\ B_{ui_1} &= -\frac{1}{2}\mathcal{F}_{i_1 j_1} x^{j_1}\,, \qquad B_{ui_2} = -\frac{1}{2}\mathcal{F}_{i_2 j_2} x^{j_2}\,, \end{aligned} \tag{3.1}$$

where B is the NSNS 2-form. We will use the same notation as in the previous section.

Worldsheet action of fundamental strings in GS formalism is

$$\begin{aligned} L = \frac{1}{\pi\alpha'} \Bigg[& \frac{1}{2}\partial_+ u \partial_- v + \frac{1}{2}\partial_+ v \partial_- u - m^2 x^i x^i + \\ & + \frac{1}{2}\mathcal{F}_{ij} x^i \left(\partial_+ u \partial_- x^j - \partial_- u \partial_+ x^j\right) + \partial_+ x^i \partial_- x^i + \partial_+ x^a \partial_- x^a + \\ & + i S_R \left(\partial_+ - \frac{1}{8}\alpha' p^u \mathcal{F}_{ij}\gamma^{ij}\right) S_R + i S_L \left(\partial_- + \frac{1}{8}\alpha' p^u \mathcal{F}_{ij}\gamma^{ij}\right) S_L \Bigg]\,, \\ = \frac{1}{\pi\alpha'} \Bigg[& \partial_+ \left(e^{2im\sigma} X\right) \partial_- \left(e^{-2im\sigma} X^\dagger\right) + \partial_+ (e^{-2im\sigma} X^\dagger) \partial_- \left(e^{2im\sigma} X\right) + \\ & + i S_R e^{2m\tau\gamma^1\gamma^2 R_+} \partial_+ \left(e^{-2m\tau\gamma^1\gamma^2 R_+} S_R\right) + i S_L e^{-2m\tau\gamma^1\gamma^2 R_+} \partial_- \left(e^{2m\tau\gamma^1\gamma^2 R_+} S_L\right) \Bigg]\,, \end{aligned} \tag{3.2}$$

where $X = \frac{1}{\sqrt{2}}(x^1 + ix^2)$ and other bosonic fields are ignored in the second expression. Though this system can also be analyzed by covariant NSR formalism [18, 19], we adopt GS formalism in order to compare with the RR background.

Two types of supersymmetry are as follows.

- dynamical supersymmetry

$$\begin{aligned}
\delta x^i &= i\epsilon_R\gamma^i R_+ S_R + i\epsilon_L\gamma^i R_+ S_L\,,\\
\delta x^a &= i\epsilon_R\gamma^i R_- S_R + i\epsilon_L\gamma^i R_- S_L\,,\\
\delta S_R &= -\partial_- x^i\epsilon_R\gamma^i R_+ - \partial_- x^a\epsilon_R\gamma^a R_- + mx^i\epsilon_R\gamma^i\gamma^1\gamma^2 R_+\,,\\
\delta S_L &= -\partial_+ x^i\epsilon_L\gamma^i R_+ - \partial_+ x^a\epsilon_L\gamma^a R_- - mx^i\epsilon_L\gamma^i\gamma^1\gamma^2 R_+\,,
\end{aligned} \tag{3.3}$$

  where $\epsilon_{R,L}R_+ = 0$.

- kinematical supersymmetry

$$\begin{aligned}
\delta S_R &= e^{2m\tau\gamma^1\gamma^2 R_+}\epsilon_R\,,\\
\delta S_L &= e^{-2m\tau\gamma^1\gamma^2 R_+}\epsilon_L\,.
\end{aligned} \tag{3.4}$$

As in the previous section, we do not add extra boundary term in this section. Then variation of the action is,

$$\begin{aligned}
\delta S = \frac{1}{\pi\alpha'}\int d^2\sigma\Bigg[&-2\delta x^{i_1}\left(\partial_+\partial_- x^{i_1} + m^2 x^{i_1} - m\epsilon_{i_1 j_1}\left(\partial_- x^{j_1} - \partial_+ x^{j_1}\right)\right) +\\
&- 2\delta x^{i_2}\left(\partial_+\partial_- x^{i_2} + m^2 x^{i_2} - m\epsilon_{i_2 j_2}\left(\partial_- x^{j_2} - \partial_+ x^{j_2}\right)\right) +\\
&+ 2i\delta s_R e^{-2m\tau\Lambda}\partial_+\left(e^{2m\tau\Lambda}s_R\right) + 2i\delta s_L e^{2m\tau\Lambda}\partial_-\left(e^{-2m\tau\Lambda}s_L\right) +\\
&+ \partial_\sigma\left\{-\frac{1}{2}\delta x^{i_1}\left(\partial_\sigma x^{i_1} - 2m\epsilon_{i_1 j_1}x^{j_1}\right) - \frac{1}{2}\delta x^{i_2}\left(\partial_\sigma x^{i_2} - 2m\epsilon_{i_2 j_2}x^{j_2}\right) -\right.\\
&\left.\qquad + \frac{i}{2}\left(S_R\delta S_R - S_L\delta S_L\right)\right\}\Bigg]\,.
\end{aligned} \tag{3.5}$$

Equations of motion are

$$\partial_+\partial_- x^{i_1} + m^2 x^{i_1} - m\epsilon^{i_1 j_1}(\partial_- x^{j_1} - \partial_+ x^{j_1}) = 0\,, \tag{3.6}$$

$$\partial_+(e^{2m\tau\Lambda}s_R) = 0\,, \tag{3.7}$$

$$\partial_-(e^{-2m\tau\Lambda}s_L) = 0\,. \tag{3.8}$$

Boundary conditions for open strings are

$$\mathrm{N}: \partial_\sigma x^{i_1} - 2m\epsilon^{i_1 j_1}x^{j_1} = 0\,, \tag{3.9}$$

$$\mathrm{D}: x^i = \text{constant}\,, \tag{3.10}$$

$$S_L = MS_R, \qquad M^T M = 1. \tag{3.11}$$

In terms of X the equation of motion and the boundary condition are

$$\partial_+\partial_-(e^{2im\sigma}X) = 0\,, \tag{3.12}$$

$$x^1,x^2 : \mathrm{N} \qquad \partial_\sigma(e^{2im\sigma}X) = 0\,, \tag{3.13}$$

$$x^1,x^2 : \mathrm{D} \qquad e^{2im\sigma}X = \text{constant}\,. \tag{3.14}$$

Thus if both x^1 and x^2 are Neumann or Dirichlet, then $e^{2im\sigma}X$ satisfy equation of motion and Neumann or Dirichlet condition in the flat background. Therefore the analysis is almost the same as in the flat background.

Mode expansion and hamiltonian of x^i is given as follows.

- x^1, x^2 : N

$$X = e^{-2im\sigma}\left[x + 2\alpha' p\tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n\left(e^{-in(\tau-\sigma)} + e^{-in(\tau+\sigma)}\right)\right], \tag{3.15}$$

$$\left[x, p^\dagger\right] = i\,, \qquad [\alpha_n, \alpha_{n'}^\dagger] = n\delta_{nn'}\,, \tag{3.16}$$

$$\begin{aligned} H &= \frac{1}{\pi\alpha'}\int_0^\pi d\sigma\left[\partial_+(e^{2im\sigma}X)\partial_+\left(e^{-2im\sigma}X^\dagger\right) + \partial_-\left(e^{2im\sigma}X\right)\partial_-\left(e^{-2im\sigma}X^\dagger\right)\right], \\ &= 2\alpha' pp^\dagger + \frac{1}{2}\sum_{n\neq 0}\left(\alpha_n\alpha_n^\dagger + \alpha_n^\dagger\alpha_n\right). \end{aligned} \tag{3.17}$$

- x^1, x^2: D

$$X = e^{-2im\sigma}\left[x_0 + (x_1e^{2im\pi} - x_0)\frac{\sigma}{\pi} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n(e^{-in(\tau-\sigma)} - e^{-in(\tau+\sigma)})\right], \tag{3.18}$$

$$[\alpha_n, \alpha_{n'}^\dagger] = n\delta_{nn'}\,, \tag{3.19}$$

where $x_0 = X(\sigma = 0), x_1 = X(\sigma = \pi)$.

$$H = \frac{1}{2\pi^2\alpha'}(x_1e^{2im\pi} - x_0)(x_1e^{2im\pi} - x_0)^* + \frac{1}{2}\sum_{n\neq 0}(\alpha_n\alpha_n^\dagger + \alpha_n^\dagger\alpha_n)\,. \tag{3.20}$$

- x^1: N, x^2 : D

$$\begin{aligned} X = e^{-2im\sigma}\Bigg[\sqrt{2}i\frac{x_0e^{2\pi im} - x_1}{e^{2\pi im} - e^{-2\pi im}} + i\sqrt{\frac{\alpha'}{2}}\sum_{n=-\infty}^{\infty}\Bigg(&\frac{1}{n+2m}\alpha_n e^{-i(n+2m)(\tau-\sigma)} + \\ &+ \frac{1}{n-2m}\alpha_{-n}^\dagger e^{-i(n-2m)(\tau+\sigma)}\Bigg)\Bigg], \end{aligned} \tag{3.21}$$

where $x_0 = x^2(\sigma = 0), x_1 = x^2(\sigma = \pi)$. (Here we assume $2m$ is not an integer. If $2m$ is integer, X has extra zero mode contribution.)

$$[\alpha_n, \alpha_{n'}^\dagger] = (n + 2m)\delta_{nn'}\,, \tag{3.22}$$

$$H = \frac{1}{2}\sum_n\left(\alpha_n\alpha_n^\dagger + \alpha_n^\dagger\alpha_n\right). \tag{3.23}$$

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Next we give mode expansions and hamiltonians of fermions. The boundary condition is

$$s_L = N s_R \qquad \text{at} \qquad \sigma = 0, \pi \,. \tag{3.24}$$

Note that $N\Lambda = \xi_1\xi_2\Lambda N$. Then,

- $\xi_1\xi_2 = -1$

$$\begin{aligned} s_L &= \sqrt{\frac{\alpha'}{2}} e^{2m\tau\Lambda} N \sum_{n=-\infty}^{\infty} s_n e^{-in(\tau+\sigma)} \,, \\ s_R &= \sqrt{\frac{\alpha'}{2}} e^{-2m\tau\Lambda} \sum_{n=-\infty}^{\infty} s_n e^{-in(\tau-\sigma)} \,, \end{aligned} \tag{3.25}$$

$$\{s_n^\alpha, s_{n'}^{\beta\dagger}\} = \delta_{nn'}\delta^{\alpha\beta}, s_n^\dagger = s_{-n} \,, \tag{3.26}$$

$$\begin{aligned} H &= \frac{i}{2\pi\alpha'} \int_0^\pi d\sigma (s_R \dot{s}_R + s_L \dot{s}_L) \\ &= \frac{1}{2} \sum_{n=-\infty}^{\infty} s_{-n}(n - 2im\Lambda) s_n \,. \end{aligned} \tag{3.27}$$

- $\xi_1\xi_2 = 1$

$$\begin{aligned} s_L &= \sqrt{\frac{\alpha'}{2}} N e^{-2m\sigma\Lambda} \sum_{n=-\infty}^{\infty} s_n e^{-in(\tau+\sigma)} \,, \\ s_R &= \sqrt{\frac{\alpha'}{2}} e^{-2m\sigma\Lambda} \sum_{n=-\infty}^{\infty} s_n e^{-in(\tau-\sigma)} \,, \end{aligned} \tag{3.28}$$

$$\{s_n^\alpha, s_{n'}^{\beta\dagger}\} = \delta_{nn'}\delta^{\alpha\beta}, s_n^\dagger = s_{-n} \,, \tag{3.29}$$

$$H = \frac{1}{2} \sum_{n=-\infty}^{\infty} n s_{-n} s_n \,. \tag{3.30}$$

Now we consider the half dynamical supersymmetry condition. Supersymmetry transformation of S_R and S_L can be rewritten as follows.

$$\begin{aligned} \delta S_R &= -\frac{1}{2}(\partial_\tau x^i - (\partial_\sigma x^i - 2m\epsilon_{i_1 j_1} x^{j_1}))\epsilon_R \gamma^i R_+ - \partial_- x^a \epsilon_R \gamma^a R_- \,, \\ \delta S_L &= -\frac{1}{2}(\partial_\tau x^i + (\partial_\sigma x^i - 2m\epsilon_{i_1 j_1} x^{j_1}))\epsilon_L \gamma^i R_+ - \partial_+ x^a \epsilon_L \gamma^a R_- \,. \end{aligned} \tag{3.31}$$

Noting that Neumann condition is given by eq.(3.9), we can see from the above form of supersymmetry transformation that BPS condition is the same as $m = 0$ case i.e. flat background, except that we have an additional condition $MR_\pm = R_\pm M$.

For example, $M = \gamma^{i_1} \ldots \gamma^{i_{p-1}}$ gives Dp-brane lying along $x^{i_1} \ldots x^{i_{p-1}}$. Note that there is no restriction for positions of D-branes.

Next let us consider half kinematical supersymmetry condition.

$$e^{2m\tau\Lambda}\kappa_L = N e^{-2m\tau\Lambda}\kappa_R \,, \tag{3.32}$$

$$\hat{\kappa}_L = \hat{N}\hat{\kappa}_R \,. \tag{3.33}$$

The first equation can be satisfied for arbitrary τ only when $\Lambda N = -N\Lambda$ i.e. $\xi_1\xi_2 = -1$. Then the amount of unbroken kinematical supersymmetry is half, otherwise 1/4.

4. D-branes in RR background with gauge flux and S-duality

In the previous two sections we have investigated boundary conditions in the RR and NSNS backgrounds respectively, without introducing additional boundary term. These backgrounds are S-dual to each other, and D3-branes in one of these background are also S-dual to D3-branes in the other background. However, we have seen that D3-branes in the RR background can sit only at $x^i = 0$ (except the case $M = \gamma^{56}$), and in the NSNS background they can be anywhere. Note that the NSNS background has non-zero NSNS B-field, which induces gauge flux on D-branes. Therefore D3-branes in section 3 are S-dual to D3-branes with non-zero gauge flux in the RR background. This means that if we turn on appropriate gauge field on D-branes in the RR background, they can move away from $x^i = 0$ without breaking any more supersymmetry. The purpose of this section is to construct such D3-branes in the RR background. Such a phenomenon has been already pointed out in [16] for pp-wave from $AdS_5 \times S^5$.

What gauge field should be turned on can be seen by noticing that S-duality is realized as standard electromagnetic duality on D3-brane effective action. The effective action in non-zero NSNS and RR 2-form background is

$$S = -\int d^4\sigma \Bigg[\sqrt{-\det(\partial_\mu X^M \partial_\nu X^N (G_{MN} + B_{MN}) + F_{\mu\nu})} \pm \\ \pm \frac{1}{4}\epsilon^{\mu\nu\lambda\rho} C_{MN} \partial_\mu X^M \partial_\nu X^N \left(F_{\lambda\rho} + B_{KL}\partial_\lambda X^K \partial_\rho X^L\right)\Bigg]. \tag{4.1}$$

To perform dual transformation, we add $\pm\int d^4\sigma \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\partial_\mu\lambda_\nu F_{\lambda\rho}$ to the above action, where λ is a Lagrange multiplier, which will be the gauge field of the dualized system. Then equation of motion obtained by varying $F_{\mu\nu}$ is

$$\partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu - C_{MN}\partial_\mu X^M \partial_\nu X^N = \pm\frac{1}{4}\epsilon_{\mu\nu\lambda\rho}\sqrt{-\det(\partial_\mu X^M \partial_\nu X^N (G_{MN} + B_{MN}) + F_{\mu\nu})} \times \\ \times \Big[(\partial_\mu X^M \partial_\nu X^N (G_{MN} + B_{MN}) + F_{\mu\nu})^{-1\lambda\rho} - \\ - (\partial_\mu X^M \partial_\nu X^N (G_{MN} + B_{MN}) + F_{\mu\nu})^{-1\rho\lambda}\Big]. \tag{4.2}$$

The left hand side of this equation is "$F + B$" of the dualized system. Therefore we can read off what gauge field must be turned on by computing the right hand side. We give results for the case where D3-branes lie along (u, v, x^1, x^2), (u, v, x^1, x^3) and (u, v, x^1, x^4).

- $(u, v, x^1, x^2)(M = \gamma^{12})$

$$F_{ui} = \pm f x^i\,. \tag{4.3}$$

  Neumann boundary condition: $\partial_\sigma x^i - F_{iu}\partial_\tau u = \partial_\sigma x^i \pm 2m x^i = 0$.

- $(u, v, x^1, x^3)(M = \gamma^{13})$

$$F_{u1} = \mp f x^4\,, \qquad F_{u3} = \pm f x^2\,. \tag{4.4}$$

Neumann boundary condition:

$$\begin{aligned}\partial_\sigma x^1 - F_{1u}\partial_\tau u &= \partial_\sigma x^1 \mp 2mx^4 = 0\,,\\ \partial_\sigma x^3 - F_{3u}\partial_\tau u &= \partial_\sigma x^3 \pm 2mx^2 = 0\,.\end{aligned}$$

- $(u, v, x^1, x^4)(M = \gamma^{14})$

$$F_{u1} = \pm f x^3\,, \qquad F_{u4} = \pm f x^2\,. \tag{4.5}$$

Neumann boundary condition:

$$\begin{aligned}\partial_\sigma x^1 - F_{1u}\partial_\tau u &= \partial_\sigma x^1 \pm 2mx^3 = 0\,,\\ \partial_\sigma x^4 - F_{4u}\partial_\tau u &= \partial_\sigma x^4 \pm 2mx^2 = 0\,.\end{aligned}$$

We can also see these gauge fluxes directly from the half dynamical supersymmetry conditions. First let us consider the case $M = \gamma^{13}$. By using $\epsilon_R M = \zeta\epsilon_L$, the supersymmetry transformation of S_R and S_L can be rewritten as follows.

$$\begin{aligned}M\delta S_R = {}&\zeta\left(\partial_- x^1 + \zeta m x^4\right)\epsilon_L\gamma^1 R_+ - \zeta\left(\partial_- x^2 - \zeta m x^3\right)\epsilon_L\gamma^2 R_+ +\\ &+\zeta\left(\partial_- x^3 - \zeta m x^2\right)\epsilon_L\gamma^3 R_+ - \zeta\left(\partial_- x^4 + \zeta m x^1\right)\epsilon_L\gamma^4 R_+\,,\end{aligned} \tag{4.6}$$

$$\begin{aligned}\delta S_L = {}&-\left(\partial_+ x^1 - \zeta m x^4\right)\epsilon_L\gamma^1 R_+ - \left(\partial_+ x^2 + \zeta m x^3\right)\epsilon_L\gamma^2 R_+ -\\ &-\left(\partial_+ x^3 + \zeta m x^2\right)\epsilon_L\gamma^3 R_+ - \left(\partial_+ x^4 - \zeta m x^1\right)\epsilon_L\gamma^4 R_+\,.\end{aligned} \tag{4.7}$$

Therefore boundary conditions are

- $\zeta = -1$

$$\partial_\sigma x^1 + 2mx^4 = 0\,, \qquad \partial_\sigma x^3 - 2mx^2 = 0\,, \qquad \partial_\tau x^2 = 0\,, \qquad \partial_\tau x^4 = 0\,.$$

- $\zeta = +1$

$$\partial_\sigma x^2 + 2mx^3 = 0\,, \qquad \partial_\sigma x^4 - 2mx^1 = 0\,, \qquad \partial_\tau x^1 = 0\,, \qquad \partial_\tau x^3 = 0\,.$$

Note that Dirichlet condition has no additional constraint such as $x^i = 0$. Therefore D-branes can be put anywhere.

We give mode expansions and hamiltonians for the following example

$$\begin{aligned}\mathrm{N} : \partial_\sigma x^1 + 2mx^4 &= 0\,,\\ \mathrm{D} : \partial_\tau x^4 &= 0\,,\end{aligned}$$

$$
\begin{aligned}
x^1 &= -\frac{x_1 - x_0 e^{-2m\pi}}{e^{2m\pi} - e^{-2m\pi}} e^{2m\sigma} + \frac{x_0 e^{2m\pi} - x_1}{e^{2m\pi} - e^{-2m\pi}} e^{-2m\sigma} + \\
&\quad + i\sqrt{\alpha'}\frac{1}{\sqrt{\omega_0}}\left(a_0^1 e^{-i\omega_0\tau} - a_0^{1\dagger} e^{i\omega_0\tau}\right) + \\
&\quad + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\left(a_n^1 e^{-i\omega_n\tau} - a_n^{1\dagger} e^{i\omega_n\tau}\right)\left(e^{in\sigma} + e^{-in\sigma}\right), \qquad (4.8)\\
x^4 &= \frac{x_1 - x_0 e^{-2m\pi}}{e^{2m\pi} - e^{-2m\pi}} e^{2m\sigma} + \frac{x_0 e^{2m\pi} - x_1}{e^{2m\pi} - e^{-2m\pi}} e^{-2m\sigma} \\
&\quad + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\left(a_n^4 e^{-i\omega_n\tau} + a_n^{4\dagger} e^{i\omega_n\tau}\right)\left(e^{in\sigma} - e^{-in\sigma}\right), \qquad (4.9)\\
\left[a_n^i, a_{n'}^{j\dagger}\right] &= \delta_{nn'}\delta^{ij}, \qquad (4.10)
\end{aligned}
$$

where $x_0 = x^4(\sigma=0), x_1 = x^4(\sigma=\pi)$.

Hamiltonian has an additional factor $\frac{1}{2\pi\alpha'}[2mx^1x^4]_{\sigma=0}^{\sigma=\pi}$ originated from the gauge field.

$$
\begin{aligned}
H &= \frac{1}{2\pi\alpha'}\int_0^{\pi} d\sigma \left[\frac{1}{2}\partial_\tau x^1\partial_\tau x^1 + \frac{1}{2}\partial_\tau x^4\partial_\tau x^4 + \frac{1}{2}\partial_\sigma x^1\partial_\sigma x^1 + \frac{1}{2}\partial_\sigma x^4\partial_\sigma x^4 + \right.\\
&\qquad \left. + \ 2m^2x^1x^1 + 2m^2x^4x^4\right] + \frac{1}{2\pi\alpha'}\left[2mx^1x^4\right]_{\sigma=0}^{\sigma=\pi}, \\
&= \frac{1}{2\pi\alpha'}\int_0^{\pi} d\sigma \left[\frac{1}{2}\partial_\tau x^1\partial_\tau x^1 + \frac{1}{2}\partial_\tau x^4\partial_\tau x^4 + \frac{1}{2}\left(\partial_\sigma x^1 + 2mx^4\right)^2 + \frac{1}{2}\left(\partial_\sigma x^4 + 2mx^1\right)^2\right], \\
&= \frac{1}{2}\left[\omega_0\left(a_0^1a_0^{1\dagger} + a_0^{1\dagger}a_0^1\right) + \sum_{n=1}^{\infty}\omega_n\left(a_n^1a_n^{1\dagger} + a_n^{1\dagger}a_n^1 + a_n^4a_n^{4\dagger} + a_n^{4\dagger}a_n^4\right)\right]. \qquad (4.11)
\end{aligned}
$$

The fermion part is not changed since its boundary condition does not change. Therefore the number of unbroken kinematical supersymmetry is the same as the case without gauge flux. Hence this D-brane is a half BPS state.

Next we investigate the case $M = \gamma^{14}$. By using $\epsilon_R M = \zeta\epsilon_L$,

$$
\begin{aligned}
M\delta S_R &= \zeta\left(\partial_- x^1 - \zeta m x^3\right)\epsilon_L\gamma^1 R_+ - \zeta\left(\partial_- x^2 - \zeta m x^4\right)\epsilon_L\gamma^2 R_+ - \\
&\quad - \zeta(\partial_- x^3 - \zeta m x^1)\epsilon_L\gamma^3 R_+ + \zeta(\partial_- x^4 - \zeta m x^2)\epsilon_L\gamma^4 R_+, \qquad (4.12)\\
\delta S_L &= -(\partial_+ x^1 + \zeta m x^3)\epsilon_L\gamma^1 R_+ - (\partial_+ x^2 + \zeta m x^4)\epsilon_L\gamma^2 R_+ - \\
&\quad - (\partial_+ x^3 + \zeta m x^1)\epsilon_L\gamma^3 R_+ - (\partial_+ x^4 + \zeta m x^2)\epsilon_L\gamma^4 R_+. \qquad (4.13)
\end{aligned}
$$

Boundary conditions are

- $\zeta = -1$

$$
\partial_\sigma x^2 + 2mx^4 = 0, \qquad \partial_\sigma x^3 + 2mx^1 = 0, \qquad \partial_\tau x^1 = 0, \qquad \partial_\tau x^4 = 0.
$$

- $\zeta = +1$

$$
\partial_\sigma x^1 - 2mx^3 = 0, \qquad \partial_\sigma x^4 - 2mx^2 = 0, \qquad \partial_\tau x^2 = 0, \qquad \partial_\tau x^3 = 0.
$$

Mode expansions and hamiltonians are similar to the previous case.

Finally let us consider the case $M = \gamma^{12}$, which is not allowed in the analysis in section 2. By $\epsilon_R M = \zeta\epsilon_L$,

$$\begin{aligned} M\delta S_R &= \zeta(\partial_- x^{i_1} - \zeta m x^{i_1})\epsilon_L\gamma^{i_1}R_+ - \zeta(\partial_- x^{i_2} + \zeta m x^{i_2})\epsilon_L\gamma^{i_2}R_+\,, &(4.14)\\ \delta S_L &= -(\partial_+ x^{i_1} + \zeta m x^{i_1})\epsilon_L\gamma^{i_1}R_+ - (\partial_+ x^{i_2} - \zeta m x^{i_2})\epsilon_L\gamma^{i_2}R_+\,. &(4.15)\end{aligned}$$

Boundary conditions are

- $\zeta = -1$

$$\partial_\sigma x^{i_1} - 2m x^{i_1} = 0\,, \qquad \partial_\tau x^{i_2} = 0\,.$$

- $\zeta = +1$

$$\partial_\tau x^{i_1} = 0, \qquad \partial_\sigma x^{i_2} - 2m x^{i_2} = 0\,.$$

Thus this case is allowed and positions of D-branes can be arbitrary.

Mode expansion of x which satisfy the boundary condition $\partial_\sigma x - 2mx = 0$ is

$$\begin{aligned} x &= \sqrt{\frac{4m\pi}{e^{4m\pi}-1}}(x_0 + 2\alpha' p\tau)e^{2m\sigma} + \\ &\quad + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\left[e^{-i\omega_n\tau}\left(\frac{n-2mi}{\omega_n}e^{in\sigma} + \frac{n+2mi}{\omega_n}e^{-in\sigma}\right)a_n - \right.\\ &\qquad\qquad \left. - e^{i\omega_n\tau}\left(\frac{n-2mi}{\omega_n}e^{in\sigma} + \frac{n+2mi}{\omega_n}e^{-in\sigma}\right)a_n^\dagger\right], \qquad (4.16)\end{aligned}$$

$$\left[a_n, a_n^\dagger\right] = \delta_{n,n'}\,, \qquad [x_0, p] = i\,. \qquad (4.17)$$

Hamiltonian has an additional factor $\frac{1}{2\pi\alpha'}[-mxx]_{\sigma=0}^{\sigma=\pi}$ from the gauge field.

$$\begin{aligned} H &= \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma\left[\frac{1}{2}\partial_\tau x\partial_\tau x + \frac{1}{2}\partial_\sigma x\partial_\sigma x + 2m^2 xx\right] + \frac{1}{2\pi\alpha'}[-mxx]_{\sigma=0}^{\sigma=\pi}\,,\\ &= \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma\left[\frac{1}{2}\partial_\tau x\partial_\tau x + \frac{1}{2}(\partial_\sigma x - 2mx)^2\right],\\ &= \alpha' p^2 + \frac{1}{2}\sum_{n=1}^{\infty}\omega_n\left(a_n a_n^\dagger + a_n^\dagger a_n\right). \qquad (4.18)\end{aligned}$$

Thus we have constructed D3-branes corresponding to D3-branes in section 3 ($M = \gamma^{12}, \gamma^{13}, \gamma^{14}, \gamma^{56}$). For $M = \gamma^{56}$ we do not have to introduce non-zero gauge field. $M = \gamma^{13}$ and γ^{14} give half BPS branes. $M = \gamma^{12}$ and γ^{56} give 1/3 BPS branes (half dynamical and 1/4 kinematical supersymmetry). It is expected that low lying open string spectra on these branes coincide with those of section 3 in $\alpha' \to 0$ limit. Indeed we can see from the form of the hamiltonians that the spectra of open strings with both ends ending on the same brane agree.

5. Conclusions

We have investigated open string boundary conditions which preserve half dynamical supersymmetry and some of kinematical supersymmetry. Section 2 is devoted to D-branes without extra boundary term in the RR background. These branes cannot move from $x^i = 0$ in most cases. Section 3 is devoted to D-branes without extra boundary term in the NSNS background. These branes can move away from $x^i = 0$. Section 4 is devoted to D-branes with gauge flux in the RR background, which correspond to those of section 2. These branes can indeed move away from $x^i = 0$ with breaking no more supersymmetry.

It is known that D-branes in pp-wave background from $AdS_5 \times S^5$ can also move away from the origin by introducing gauge field [16]. Similar phenomenon might happen in other pp-wave backgrounds. It is interesting to investigate it by the same method as this paper or Born Infeld action as in [16].

In [7] it is shown that in the case of pp-wave from $AdS^5 \times S^5$ only half BPS D-branes satisfy the Cardy condition and therefore less supersymmetric D-branes are inconsistent. Similarly less supersymmetric D-branes constructed in this paper might be shown to be inconsistent by checking the Cardy condition.

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References

[1] R.R. Metsaev, *Type-IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[2] R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[3] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. and Quant. Grav.* **19** (2002) L87 [hep-th/0201081];
M. Blau, J. Figueroa-O'Farrill and G. Papadopoulos, *Penrose limits, supergravity and brane dynamics*, *Class. and Quant. Grav.* **19** (2002) 4753 [hep-th/0202111].

[4] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[5] M. Billó and I. Pesando, *Boundary states for gs superstrings in an HPP wave background*, *Phys. Lett.* **B 536** (2002) 121 [hep-th/0203028].

[6] A. Dabholkar and S. Parvizi, *Dp-branes in PP-wave background*, *Nucl. Phys.* **B 641** (2002) 223 [hep-th/0203231].

[7] O. Bergman, M.R. Gaberdiel and M.B. Green, *D-brane interactions in type-IIB plane-wave background*, hep-th/0205183.

[8] C.-S. Chu and P.-M. Ho, *Noncommutative D-brane and open string in PP-wave background with B-field*, *Nucl. Phys.* **B 636** (2002) 141 [hep-th/0203186].

[9] C.-S. Chu, P.-M. Ho and F.-L. Lin, *Cubic string field theory in PP-wave background and background independent moyal structure*, *J. High Energy Phys.* **09** (2002) 003 [hep-th/0205218].

[10] P. Lee and J.-w. Park, *Open strings in PP-wave background from defect conformal field theory*, hep-th/0203257.

[11] D. Berenstein, E. Gava, J.M. Maldacena, K.S. Narain and H. Nastase, *Open strings on plane waves and their Yang-Mills duals*, hep-th/0203249.

[12] V. Balasubramanian, M.-x. Huang, T.S. Levi and A. Naqvi, *Open strings from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **08** (2002) 037 [hep-th/0204196].

[13] J.G. Russo and A.A. Tseytlin, *On solvable models of type-IIB superstring in NS-NS and RR plane wave backgrounds*, *J. High Energy Phys.* **04** (2002) 021 [hep-th/0202179].

[14] A. Kumar, R.R. Nayak and Sanjay, *D-brane solutions in PP-wave background*, *Phys. Lett.* **B 541** (2002) 183 [hep-th/0204025].

[15] M. Alishahiha and A. Kumar, *D-brane solutions from new isometries of PP-waves*, *Phys. Lett.* **B 542** (2002) 130 [hep-th/0205134].

[16] K. Skenderis and M. Taylor, *Branes in AdS and PP-wave spacetimes*, *J. High Energy Phys.* **06** (2002) 025 [hep-th/0204054].

[17] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, vol. 1, Cambridge University Press, Cambridge 1987.

[18] H. Takayanagi and T. Takayanagi, *Open strings in exactly solvable model of curved space-time and PP-wave limit*, *J. High Energy Phys.* **05** (2002) 012 [hep-th/0204234].

[19] Y. Hikida and Y. Sugawara, *Superstrings on PP-wave backgrounds and symmetric orbifolds*, *J. High Energy Phys.* **06** (2002) 037 [hep-th/0205200].

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